Exact Evaluation of Stability Margin of Multiloop Flight Control Systems

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Introduction

MULTILOOP flight control systems are widely used in the advanced aircraft in recent years. Such highly augmented aircraft have considerable potential in terms of flying qualities. There are many design methods for the multiloop control system, but a method to exactly evaluate the stability margin for multiloop control systems has not been established yet. It is very important for the designers to exactly know the stability margin of the flight control system because it directly comes to bear on flight safety.

Mukhopadhyay and Newsom[1] studied the evaluation of multiloop stability margin using singular values. In their study, square diagonal matrix $L = \mathrm{Diag}[K_n \exp(j\phi_n)]$ was introduced at the plant input to determine the range in which both K_n and ϕ_n can be changed simultaneously in all loops for which the system would remain stable. And, the relation between the singular value of the system return difference matrix $\sigma(I + FG)$ and the simultaneous gain and phase change $[K_n, \phi_n]$ was illustrated. But, predictions based on the singular value are generally conservative. To improve the conservatism of these predictions, Ly[2] proposed a method using the eigenvalue norm $|1 + \lambda[FG]|$ as a measure of robustness, and Yeh et al.[3] proposed a method using the eigenvalue magnitude of the system return difference matrix $|\lambda[I + FG]|$. But, these methods cannot be used to exactly evaluate the stability margin.

In this Note, analysis methods for the exact evaluation of the stability margin of multiloop flight control systems are presented. The minus inverse vector ξ and the open-loop transfer function for the multiloop flight control system $-1/\xi$ vector are introduced. Both methods are the extension of that for single-loop control systems. Using these methods, the gain and phase margins of the system can be considered individually.

Stability Margin of Single-Loop Control Systems

First, the classical single-loop Nyquist test is considered for stability margin evaluation. Let the open-loop transfer function W be as follows:

$$W(S) = Q(S)/P(S) \tag{1}$$

Then, the response of the closed loop can be expressed as

$$\frac{x}{uc} = \frac{W(s)}{1 + W(s)} \tag{2}$$

So, the stability margin of single-loop control systems can be evaluated to measure the proximity of the vector locus of $W(j\omega)$ to the -1.0 point.

For example, if the following open-loop transfer function is considered

$$\frac{Q}{P} = \frac{(s+8)}{(s+1)(s+2)} \tag{3}$$

then the vector locus of the open-loop transfer function of Eq. (3) is shown in Fig. 1. It follows that the evaluation of the stability margin of the single-loop control system can be carried out by inserting the complex number ξ of Eq. (4) at the plant input u shown in Fig. 2 by solving the characteristic equation of Eq. (5):

$$\xi \equiv K e^{j\lambda} = \mu + j\nu \tag{4}$$

$$1 + \xi(j\omega) \cdot W(j\omega) = 0 \tag{5}$$

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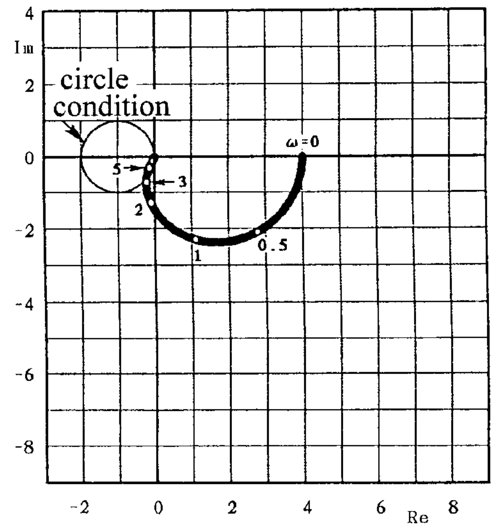


Fig. 1 Stability margin of single loop.

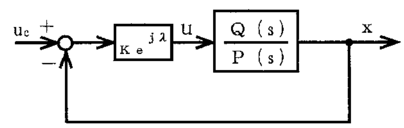


Fig. 2 System introduced ξ at the plant input u.

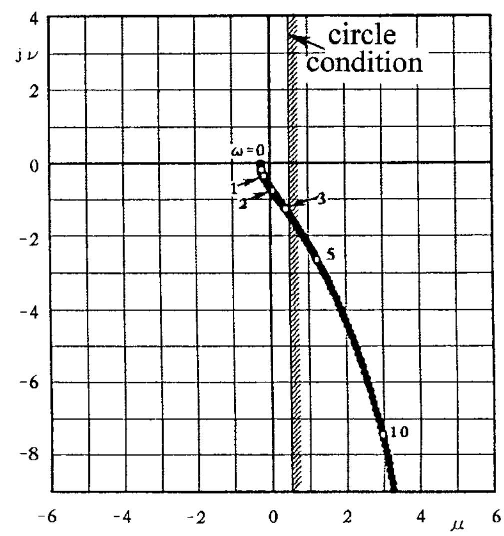


Fig. 3 ξ locus corresponding to Fig. 1.

Therefore, ξ is obtained as follows:

$$\xi = -1/W(j\omega) \tag{6}$$

This ξ is referred to as the minus inverse vector in this Note because it is expressed as a negative value of the inverse of the open-loop transfer function W. The ξ locus for Fig. 1 is shown in Fig. 3. When ξ is inserted at the plant input u, the system becomes unstable. Therefore, the ξ locus is a direct indication of the stability margin of the control system.

Now, $-1/\xi$ from the minus inverse vector ξ is defined as

$$-1/\xi = W(j\omega) \tag{7}$$

Therefore, $-1/\xi$ corresponds to the open-loop transfer function W in the single-loop control system.

Stability Margin of Multiloop Control Systems

Now, a multiloop feedback control system is considered. Let the aircraft dynamics be as follows:

$$G(S) = (SI - A)^{-1}B \tag{8}$$

Then, the response of the closed loop is expressed as

$$x = GT_2^{-1}uc = (SI - A + BF)^{-1}Buc \tag{9}$$

where F is the feedback gain matrix. The system return difference matrix T_2 is defined as

$$T_2 \equiv I + FG \tag{10}$$

Also, the characteristic equation of the closed system is given by

$$\det(T_2) = 0 \tag{11}$$

An example of a lateral-directional flight control system, which has two-input (aileron and rudder) system, shall now be considered. To evaluate the stability margin for the multiloop control system, the minus inverse vector ξ and the open-loop transfer function $-1/\xi$ are introduced. They are the extension of that for the single-loop control system. And ξ for the two-input system is inserted at each plant input. The stability margin of multiloop control systems investigated in this Note is for the case of simultaneous gain and phase change in all loops.[1] This hypothesis is correct for conventional flight-control systems because the effectiveness of all control surfaces varies with flight conditions, whereas delays in digital computation and aerodynamic phenomena affect all control surfaces concurrently.

First, the characteristic equation for the N-input system is derived from Eq. (11) using each feedback gain F multiplying ξ as follows:

$$\det(I + \xi FG) = \begin{vmatrix} 1 + \xi h_{11} & \xi h_{12} & \cdots & \xi h_{1N} \\ \xi h_{21} & 1 + \xi h_{22} & \cdots & \xi h_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \xi h_{N1} & \xi h_{N2} & \cdots & 1 + \xi h_{NN} \end{vmatrix}$$

$$= 1 + \xi \frac{Q}{P} + \xi^2 \frac{R_2}{P} + \cdots + \xi^{N-1} \frac{R_{N-1}}{P} + \xi^N \frac{R_N}{P} = 0 \tag{12}$$

where $h_{ij} = (FG)_{ij}$ is the element of matrix (FG) and

$$\frac{Q}{P} = \mathrm{tr}(FG) \tag{13}$$

$$\frac{R_M}{p} = \sum_{(i,j,\ldots,u)} A_{ii,jj,\ldots,uu} \tag{14}$$

where the minor determinant $A_{ii,jj,\ldots,uu}$ is given by all combinations consisting of $(N - M)$ in number of N:

$$R_{N-1}/P = A_{11} + A_{22} + \cdots + A_{NN} \tag{15}$$

$$R_N/P = \det(FG) \tag{16}$$

From Eq. (12) the characteristic equation of $-1/\xi$ is derived as follows:

$$(1/\xi)^N + (Q/P)(1/\xi)^{N-1} + (R_2/P)(1/\xi)^{N-2} + \cdots$$
$$+ (R_{N-1}/P)(1/\xi) + R_N/P = 0 \tag{17}$$

The $-1/\xi$ locus of N-input system is given by solving Eq. (17). N loci are obtained in the N-input multiloop system. The $-1/\xi$ locus can be interpreted as the eigenvalue of the matrix (FG) from Eq. (12) as follows:

$$\det(I + \xi FG) = (-\xi)^N \det[(-1/\xi)I - FG] = 0 \tag{18}$$

Now the characteristic equation of $-1/\xi$ for the two-input system ($N = 2$) can be expressed as

$$(1/\xi)^2 + (Q/P)(1/\xi) + R_2/P = 0 \tag{19}$$

where

$$Q/P = \mathrm{tr}(FG) = (FG)_{11} + (FG)_{22}$$

$$R_2/P = \det(FG) = (FG)_{11}(FG)_{22} - (FG)_{12}(FG)_{21} \tag{20}$$

Therefore, from Eq. (19) the $-1/\xi$ to evaluate the stability margin for the two-input system are obtained as follows:

$$-1/\xi = Q/2P \pm [(Q/2P)^2 - R_2/P]^{\frac{1}{2}} \tag{21}$$

Example

A lateral-directional flight-control system is considered, where the aircraft dynamics is shown from Ref. 4 as follows:

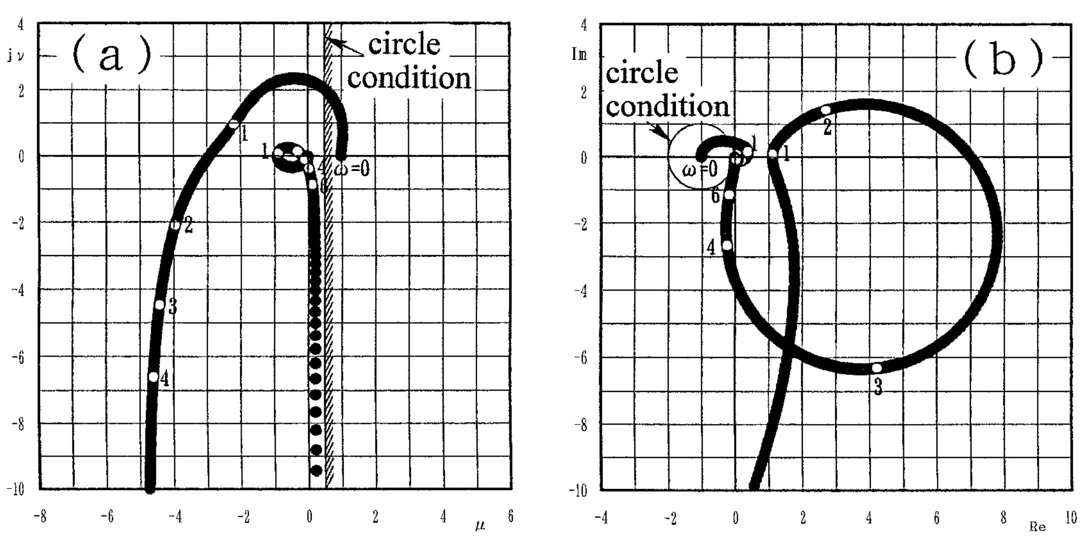
$$A = \begin{pmatrix} -0.277 & 0 & -1 & 0.035 \\ -27.6 & -1.89 & 2.59 & 0 \\ 7.50 & -0.044 & -0.627 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0.039 \\ -71.2 & 11.2 \\ -9.34 & -3.28 \\ 0 & 0 \end{pmatrix} \tag{22}$$

Locations of the poles and zeros of the closed loop ϕ/uc, which is the bank-angle response to lateral control, are given using feedback control as follows:

$$\text{poles: } -3.0 \pm j3.0,\ -2.50,\ 0.0, \qquad \text{zeros: } -3.0 \pm j3.0 \tag{23}$$

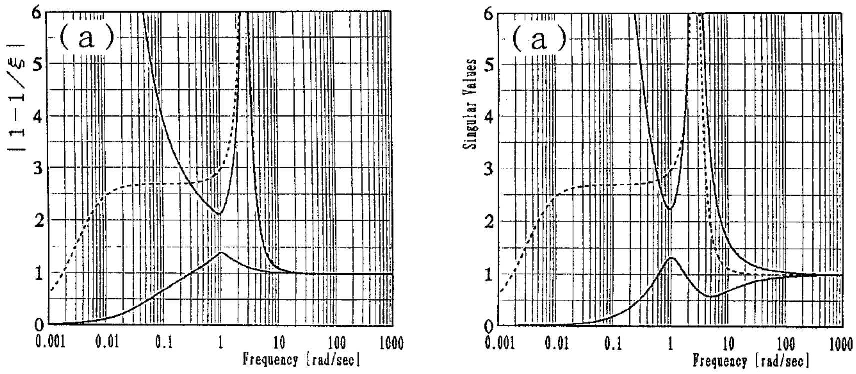
Then, the ξ locus and the $-1/\xi$ locus are shown in Figs. 4a and 4b. Both of them are expressed by the two loci. The phase margin at $\omega = 6.5$ rad/s is 81.3 deg.



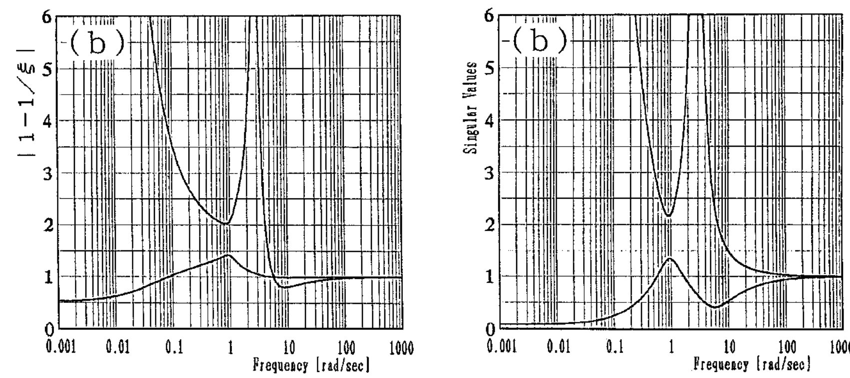
a) ξ vector locus

b) $-1/\xi$ vector locus

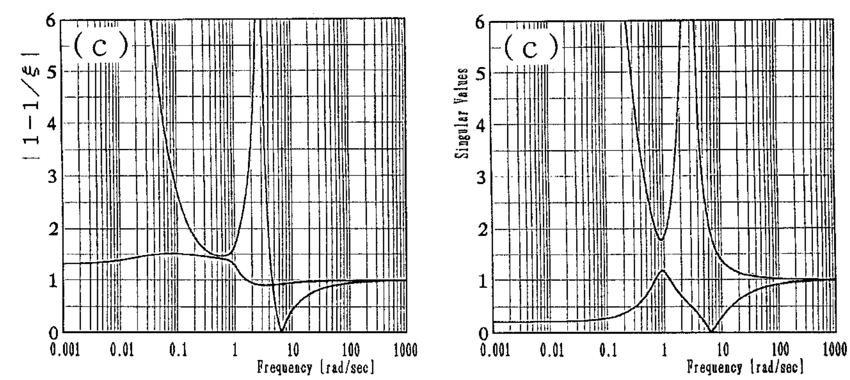
Fig. 4 Lateral-directional flight control system.



a) Nominal (corresponding to Fig. 4)



b) Case where phase is delayed by 30 deg



c) Case where phase is delayed by 81.3 deg

Fig. 5 Comparison of $|1 - 1/\xi|$ and $\sigma(I + FG)$.

Next, the relation between the present method and the method using the singular value is investigated. In case of a two-input system, the following relation is derived:

$$|1 - 1/\xi_1| \cdot |1 - 1/\xi_2| = \sigma_1(I + FG) \cdot \sigma_2(I + FG) \qquad (24)$$

where $|1 - 1/\xi|$ is the distance between the $-1/\xi$ locus and the -1.0 point. The $|1 - 1/\xi|$ and $\sigma(I + FG)$ corresponding to Fig. 4 are shown in Fig. 5a. The broken line in Fig. 5a shows the result of the equation multiplied by two values as shown in Eq. (24). Figure 5b shows $|1 - 1/\xi|$ and $\sigma(I + FG)$ in the case where the phase is delayed by 30 deg. Figure 5c shows the case where the phase is delayed by 81.3 deg, then the system becomes unstable. This corresponds to $|1 - 1/\xi|$ and $\sigma(I + FG)$ coming to zero at $\omega = 6.5$ rad/s.

At any rate, both $|1 - 1/\xi|$ and $\sigma(I + FG)$ methods are inadequate to evaluate the stability margin for multiloop flight control systems because the phase effect is not taken into consideration. On the other hand, $-1/\xi$ locus method is a very useful design tool to consider gain and phase margins of the system individually.

Conclusions

Analysis methods for the exact (i.e., nonconservative) evaluation of the stability margin of multiloop flight control systems are presented by extending the method for single-loop control systems. Those are the minus inverse vector ξ and the vector locus $-1/\xi$. When the complex gain ξ is concurrently inserted at each plant input, the system becomes unstable. The $-1/\xi$ is considered as an open-loop transfer function of multiloop control systems because of the similarity to single-loop control systems. The $-1/\xi$ is identical to the eigenvalues of the loop transfer matrix. The relation between the $-1/\xi$ method and the method using the singular value is investigated. The $-1/\xi$ method is a very useful design tool to individually consider gain and phase margins of the system.

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Large-Scale Air Combat Tactics Optimization Using Genetic Algorithms

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I. Introduction

THE complete design and specification of air combat tactics for many-vs-many engagements poses a considerable challenge to tactical planners. Although solutions have been developed for one-vs-one or few-vs-few encounters,[1] the results may not generalize to larger engagements where formation tactics become increasingly important. Conventional methods such as trajectory optimization rely on gradient information that may not be available when the free parameters are discrete quantities such as the number of aircraft, weapons type, etc. Previous studies have examined the development of optimized air combat maneuvering via optimal control and machine learning methods.[2] However, the advent of high-speed, low-cost microprocessors has made feasible the use of genetic algorithms (GAs) for solving optimization problems where gradient information is unavailable. GAs have shown considerable potential in solving optimization problems that defy conventional approaches.[3]

In this Note an approach to large-scale air combat tactics optimization using genetic algorithms (ACTOGA) and the results obtained using the approach are presented. A prototype modeling tool has been developed, integrating a tactical engagement simulator with a GA "engine" for performance-based optimization of blue team tactics. The engagement simulator uses point-mass representations of aircraft dynamics, and it allows the simulation of forward-quarter beyond-visual-range (BVR) intercepts.[1] The goal of a BVR intercept is to maneuver one's aircraft to place a threat that is not visible to the naked eye (but visible to onboard sensors such as radar) within the aircraft's weapons envelope and to then launch medium- to long-range missiles against that threat. Only the initial launch of BVR missiles is optimized by the GA; the optimization of close-in "dogfighting" or defensive maneuvering is beyond the scope of this study. The outcome of an engagement (kills, losses, etc.) is used to guide the evolution of the optimized air combat tactics. A novel feature of the approach described here is the use of a "formation hierarchy" in which small, well-known conventional fighting units are aggregated to build large-scale tactical formations. This approach facilitates the design of tactics compatible with existing air combat principles. Excellent blue team performance is demonstrated where both sides are matched in terms of formation size and aircraft capabilities.

II. Air Combat Tactics Optimization

The term *air combat tactics* encompasses several concepts, including 1) the individual maneuvers that pilots use for accomplishing a given objective, 2) formation tactics that specify how small groups of aircraft can work cooperatively, and 3) principles for constructing division tactics that integrate large groups of aircraft.

When seeking to optimize air combat tactics, the objectives must be specified unambiguously. Optimizing individual maneuvers essentially amounts to optimal control design. However, optimal paths for a given situation do not provide any guidance on how to construct effective fighting groups for an arbitrary MvN engagement. Furthermore, there is no way to ensure that a pilot can execute a given optimal maneuver or remember n different optimal maneuvers for n different situations. As such, the approach taken here is to use conventional tactical maneuvers and optimize the manner in which they are used by a large aircraft formation.

The most effective formation tactics employ a basic fighting unit of two aircraft (called a *section* or *element*).[1] Because this is how all fighter pilots learn their craft, it was determined early in this research that it would be most effective for optimized tactics not to deviate from established tactical doctrine. Accordingly, software tactics modules that employ basic fighting units of two aircraft have been developed. Because large numbers of fighter aircraft are difficult to control, formation tactics for large groups can be developed using a hierarchical structure consisting of smaller units or divisions, for example, a four-airplane division called the *fluid four* consists of two elements. Each element consists of two aircraft, but they are treated as a unit. This hierarchical concept is used to develop a GA-based approach to optimized air combat tactics development. Given a palette of air combat maneuvers and standard small-formation tactics as building blocks, GAs are used to determine how they can be integrated to produce large fighting groups that optimize overall combat effectiveness.

Tactics implementation in ACTOGA proceeds as follows:

1) Define a set of commonly used element and division formations (Table 1) as well as the underlying tactical maneuvers and attack tactics.

2) Develop a set of principles for aggregating the small formation tactics for large MvN engagements and implement a method for doing so in the GA software. To illustrate, consider a team consisting of four aircraft. Using only the fighting wing and double attack, the possible team formations are shown in Fig. 1 (assuming both elements use the same two-ship formation). A similar approach can be used to develop large-division formations from smaller two-ship and four-ship groupings.

3) Use the resultant formation tactics to drive the engagement and evaluate the results via the performance metric generator.

4) Optimize the MvN engagement tactics with respect to the performance metrics.

Table 1 Commonly used formations[1]

Name	Symbol	Number of entities
Fighting wing	FW	2
Left double attack	DAL	2
Right double attack	DAR	2
Fighting wing	FW	2
Finger four	FF	4
Left sections in trail	LST	4
Right sections in trail	RST	4
Wall formation	WF	4

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